Theory of Magnetostatic Waves in Moving Ferrite Films and Applications to Rotation Rate Sensing

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Abstract —A first-order field theory for electromagnetic waves in moving ferrites and ferrite thin films is presented. The dominant effect of the motion is found to be the Doppler-shifted frequency observed in the moving frame. This gives rise to an anomalously large shift in wavenumber owing to the dispersive nature of the ferrite medium. Because of the large effect, it is suggested that a moving medium experiment using magnetostatic waves could be used to distinguish between various competing forms for the dispersion term in the Fresnel–Fizeau coefficient.

The large Fresnel-Fizeau coefficient suggests that magnetostatic waves could be used to measure relative rotation rates if confined to propagate around the perimeter of a rotating disk. Since the phase shift would be established in the time required to propagate around the disk, the response time could be significantly shorter than conventional tachometers.

An experiment with counterpropagating magnetostatic waves would clarify the effect of a magnetic medium on the magnitude of the Sagnac effect. Although it should be possible, in principle, to construct an absolute rotation rate sensor using magnetostatic waves (or more precisely, magnetic polaritons), the magnitude of the Sagnac effect is predicted to be the same as for ordinary electromagnetic waves with the same frequency. Since the magnitude of the Sagnac phase shift is proportional to frequency, optical interferometers are still preferable.

I. Introduction

THE USE OF acoustic and magnetostatic surface waves for rotation rate sensing was first proposed by Newburgh *et al.* [1]. For such an application, the waves would be guided around a circular path on a rotating medium. The motion of the medium would alter the propagation velocity of the guided waves, thus causing a phase shift proportional to the rotation rate. Their analysis assumed that the effects of medium motion on both types of waves could be described in terms of Galilean velocity addition for nonrelativistic velocities. To evaluate this assumption, a theory of electromagnetic waves in moving ferrites is required.

If the medium is moving with velocity V relative to an observer A, then the phase velocity of an electromagnetic wave in the medium as measured by A is

$$u = u_0 + \alpha_{FF} V \tag{1}$$

where u_0 is the phase velocity measured by A when $V = 0$ and α_{FF} is the Fresnel–Fizeau drag coefficient. For a dispersive medium, the coefficient α_{FF} can be written [2]

$$\alpha_{FF}(\omega) = 1 - \left[\frac{1}{n(\omega)}\right]^2 + \frac{\omega}{n(\omega)}\frac{\partial n}{\partial \omega} \tag{2}$$

where ω is the angular frequency measured by A, the refractive index n is defined by

$$n(\omega) \equiv \frac{c}{u_0(\omega)} \tag{3}$$

and c is the velocity of light in free space. Although derived in the context of the propagation of light in isotropic dielectrics, (2) is also valid for anisotropic media at microwave frequencies if (3) is used to define an effective index of refraction. This point is emphasized in Section II, where an alternative but equivalent expression for α_{FF} is derived in terms of wave phase and group velocities rather than an index of refraction.

The last term in (2) is present only in dispersive media. Einstein [2] pointed out that the presence of such a term is due to the Doppler frequency shift caused by the motion of the medium. The form of this term may vary, however, depending on the specific geometry under consideration. Lerche [3] has also pointed out that several forms of the dispersion term have appeared in the literature, and that available experimental data are not precise enough to distinguish between the various forms. These data are based on light propagating in water. Since water is not very dispersive at optical frequencies, the effects of the dispersion term are small and thus difficult to obtain with sufficient accuracy. We shall show that for magnetostatic waves the dispersion term is large and dominates α_{FF}. Thus, in addition to applications to motion sensing, measurement of the Fresnel–Fizeau effect for magnetostatic waves may permit a fundamental verification of the form of the dispersion term.

The general problem of electromagnetic waves in moving anisotropic media has been considered by several authors [4]–[7]. In Section III, we adapt the key features of these theories to the case of moving ferrites and the guided modes of thin ferrite films. The results of this analysis are found to be consistent with the conclusions of Section II. The magnetostatic limit of the electromagnetic theory is taken in Section IV, and applications to relative and

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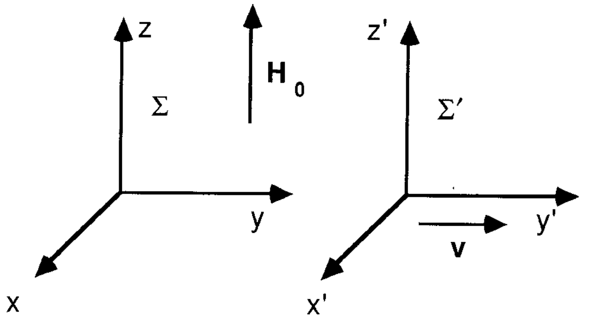

Fig. 1. Geometry and coordinate systems for analysis of electromagnetic waves in moving ferrites.

absolute rotation rate sensing are discussed in Sections V and VI.

II. Fresnel–Fizeau Coefficient for Dispersive Media

Consider a source with frequency ω in frame Σ which launches an electromagnetic wave in a medium which is moving with velocity V relative to Σ (Fig. 1). The frame in which the medium is stationary is Σ'. The laws of transformation for frequency ω and wavenumber β between the two systems is (assuming $V \| \boldsymbol{\beta}$)

$$\omega' = \gamma(\omega - \beta V) \qquad \omega = \gamma(\omega' + \beta' V) \tag{4}$$

$$\beta' = \gamma\left[\beta - \frac{\omega V}{c^2}\right] \qquad \beta = \gamma\left[\beta' + \frac{\omega' V}{c^2}\right] \tag{5}$$

where $\gamma = [1-(V/c)^2]^{-1/2}$. The phase velocities in the two systems are $u = \omega/\beta$ and $u' = \omega'/\beta'$:

$$u' = \frac{\omega - \beta V}{\beta - \dfrac{\omega V}{c^2}} \approx u - V\left[1-(u/c)^2\right] \tag{6}$$

and

$$u \approx u' + V\left[1-(u'/c)^2\right] \tag{7}$$

to first order in V/c.

Let us define the phase velocity function for a stationary medium as $u_0(\omega)$. For an observer in Σ', this function should apply, though the frequency will be Doppler shifted. Thus we can write

$$\begin{aligned} u' &= u_0(\omega') \\ &\approx u_0(\omega) + \left.\frac{\partial u_0}{\partial \omega}\right|_{\omega} \Delta\omega \end{aligned} \tag{8}$$

where $\omega' = \omega + \Delta\omega$, and

$$\Delta\omega = -\frac{\omega V}{u_0(\omega)} + O(V^2). \tag{9}$$

To obtain the partial derivative in (8), we note that the reciprocal group velocity can be written

$$\frac{1}{u_{g0}} = \frac{1}{u_0} - \frac{\omega}{u_0^2}\frac{\partial u_0}{\partial \omega}. \tag{10}$$

Solving this expression for $\partial u_0/\partial \omega$ and substituting the result along with (9) into (8) gives

$$u' = u_0(\omega) - V\left[1 - \frac{u_0(\omega)}{u_{g0}(\omega)}\right] \tag{11}$$

to first order in V. Substituting this result into (7) and collecting terms yields the result

$$u = u_0(\omega) + V\left[1 - \left[\frac{u_0(\omega)}{c}\right]^2 - \left[1 - \frac{u_0(\omega)}{u_{g0}(\omega)}\right]\right]. \tag{12}$$

The coefficient α_{FF} can be obtained by comparing (12) and (1):

$$\alpha_{\mathrm{FF}} = 1 - \left[\frac{u_0(\omega)}{c}\right]^2 - \left[1 - \frac{u_0(\omega)}{u_{g0}(\omega)}\right]. \tag{13}$$

This expression is equivalent to (2) but is expressed in terms of phase and group velocities rather than the index of refraction. Here the last bracketed term is due to dispersion and vanishes when the phase and group velocities are equal. This result is valid for anisotropic as well as isotropic media provided that the phase and group velocities are parallel or antiparallel and that the direction of the group velocity does not change with frequency.

III. Surface Waves in Moving Ferrite Thin Films

A. First-Order Minkowski Constitutive Relations

The Minkowski constitutive relations for moving anisotropic medium are of the bianisotropic form [4], [5]

$$\boldsymbol{D} = \bar{\epsilon}\cdot\boldsymbol{E} + \bar{\xi}\cdot\boldsymbol{H} \tag{14}$$

$$\boldsymbol{B} = \bar{\zeta}\cdot\boldsymbol{E} + \bar{\mu}\cdot\boldsymbol{H} \tag{15}$$

where $\bar{\xi}^{\dagger} = \bar{\zeta}$ if the medium is lossless. In general, it may be possible to decompose the total field vectors into static and time-varying components. Equations of the form of (14) and (15) may be written for both components, although the static and dynamic constitutive tensors will generally be different. For the case of an electrically isotropic ferrite, the total magnetic field intensity inside the ferrite is the sum of the static bias field $\boldsymbol{H}_0$ and the dynamic wave field $\boldsymbol{H}$:

$$\begin{aligned} \boldsymbol{H}_T &= \boldsymbol{H}_0 + \boldsymbol{H}(t) \\ &= H_0\hat{\boldsymbol{b}} + \boldsymbol{H}(t). \end{aligned} \tag{16}$$

The dynamic constitutive tensors to first order in V/c can be written [4], [8]

$$\bar{\epsilon} = \epsilon'\bar{\boldsymbol{I}} = \epsilon_0\epsilon_r'\bar{\boldsymbol{I}} \tag{17}$$

$$\bar{\mu} = \mu_0\bar{\mu}_r' = \mu_0\left[(1+\chi')\bar{\boldsymbol{I}} + i\kappa'\hat{\boldsymbol{b}}\times\boldsymbol{I} - \chi'\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\right] \tag{18}$$

$$\bar{\zeta} = \frac{1}{c^2}\left[\bar{\boldsymbol{I}} - \epsilon_r'\bar{\mu}_r'\right]\cdot\left[\boldsymbol{V}\times\bar{\boldsymbol{I}}\right] \tag{19}$$

$$\bar{\xi} = -\frac{1}{c^2}\left[\boldsymbol{V}\times\bar{\boldsymbol{I}}\right]\cdot\left[\bar{\boldsymbol{I}} - \epsilon_r'\bar{\mu}_r'\right] = \bar{\zeta}^{\dagger}. \tag{20}$$

Here the primes denote quantities in the moving frame Σ', and the unprimed quantities apply to frame Σ, which is at rest with respect to the source. The susceptibility tensor elements χ' and κ' are

$$\chi' = \frac{\omega_0'\omega_M'}{(\omega_0')^2 - (\omega')^2} \tag{21}$$

$$\kappa' = \frac{\omega'\omega_M'}{(\omega_0')^2 - (\omega')^2} \tag{22}$$

where $\omega_0' = \gamma_g'\mu_0 H_0'$, $\omega_M' = \gamma_g'\mu_0 M_s'$, and $\gamma_g' = |e'|/m_e'$ is the gyromagnetic ratio. To rigorously relate the primed quantities to the unprimed quantities we must use the relativistic transformations for fields, masses, volume (Lorentz–Fitzgerald contraction effect), and spin [9]. However, to first order in V/c, the only effect is a Doppler shift in frequency. Thus, for nonrelativistic velocities we have

$$\omega_0' = \omega_0 \tag{23}$$

$$\omega_M' = \omega_M \tag{24}$$

$$\omega' = \omega - \beta V. \tag{25}$$

In contrast, we assume the permittivity is constant with frequency so that $\epsilon_r' = \epsilon_r$.

B. Dispersion Relation for an Infinite Ferrite with $\boldsymbol{V}$, $\boldsymbol{k}$ *Perpendicular to* $\boldsymbol{H}_0$

Substituting (14) and (15) into Maxwell's curl equations and eliminating $\boldsymbol{H}$ leads to the wave equation in $\boldsymbol{E}$ for a bianisotropic medium:

$$\overline{\boldsymbol{W}}\cdot\boldsymbol{E} = \boldsymbol{0} \tag{26}$$

where the wave matrix $\overline{\boldsymbol{W}}$ is given by [5]

$$\overline{\boldsymbol{W}} = \left[\omega\bar{\xi} + \boldsymbol{k}\times\bar{\boldsymbol{I}}\right]\cdot\bar{\mu}^{-1}\cdot\left[\omega\bar{\zeta} - \boldsymbol{k}\times\bar{\boldsymbol{I}}\right] - \omega^2\bar{\epsilon} \tag{27}$$

and the field is assumed to have the spatial dependence $\exp(i\boldsymbol{k}\cdot\boldsymbol{r})$. Substituting (17) for an isotropic dielectric and keeping only first-order terms in V/c gives

$$\begin{aligned}\overline{\boldsymbol{W}} \approx \omega\left[(\boldsymbol{k}\times\bar{\boldsymbol{I}})\cdot\bar{\mu}^{-1}\cdot\bar{\zeta} - \bar{\xi}\cdot\bar{\mu}^{-1}\cdot(\boldsymbol{k}\times\bar{\boldsymbol{I}})\right]\\ -(\boldsymbol{k}\times\bar{\boldsymbol{I}})\cdot\bar{\mu}^{-1}\cdot(\boldsymbol{k}\times\bar{\boldsymbol{I}}) - \omega^2\epsilon'\bar{\boldsymbol{I}}.\end{aligned} \tag{28}$$

The dispersion relation is now obtained by setting the determinant of the wave matrix to zero:

$$\det\overline{\boldsymbol{W}} = 0. \tag{29}$$

Let us consider a nonuniform plane wave propagating parallel to the medium velocity but perpendicular to the static magnetic bias field. For definiteness we will assume the following coordinate system (Fig. 1):

$$\begin{aligned}\hat{\boldsymbol{b}} &= \hat{z}\\ \boldsymbol{V} &= V\hat{y}\\ \boldsymbol{k} &= -is\alpha\hat{x} + \beta\hat{y}, \qquad s = \pm 1.\end{aligned} \tag{30}$$

The choice of the sign of s allows consideration of nonuniform plane waves either growing or decaying along the $+x$ axis, while β is positive-definite. Evaluating the determinant (29) for this specific case leads to two roots, corresponding to the ordinary and extraordinary waves. The dispersion relations for both waves are of the form

$$\alpha^2 = \beta^2 - \frac{\omega^2\epsilon_r\mu_{\text{eff}}}{c^2} - \frac{2\omega\beta V}{c^2}\left[1 - \epsilon_r\mu_{\text{eff}}\right] \tag{31}$$

where μ_{eff} is an effective relative permeability given by

$$\mu_{\text{eff}} = \begin{cases} 1 & \text{for the ordinary wave}\\ \dfrac{(1+\chi')^2 - (\kappa')^2}{(1+\chi')} & \text{for the extraordinary wave.}\end{cases} \tag{32}$$

Recall that when evaluating χ' and κ', the Doppler-shifted frequency $\omega' = \omega - \beta V$ must be used. Because of this, (31) contains higher order terms in V than is explicitly shown. The higher order terms can be removed, but at the expense of complexity. Consistently retaining only first-order terms in V gives

$$\alpha^2 = \beta^2 - \frac{\omega^2\epsilon_r\mu_{\text{eff}}^{(0)}}{c^2} - \frac{2\omega\beta V}{c^2}\left[1 - \epsilon_r\mu_{\text{eff}}^{(0)} + \frac{\epsilon_r\mu_{\text{eff}}^{(1)}}{2}\right] \tag{33}$$

where

$$\mu_{\text{eff}}^{(0)} = \frac{(1+\chi_0)^2 - \kappa_0^2}{1+\chi_0} \tag{34}$$

$$\mu_{\text{eff}}^{(1)} = \frac{\chi_1(1+\chi_0)^2 - 2\kappa_0\kappa_1(1+\chi_0) + \chi_1\kappa_0^2}{(1+\chi_0)^2} \tag{35}$$

and

$$\chi_0 = \frac{\omega_0\omega_M}{\omega_0^2 - \omega^2} \tag{36}$$

$$\kappa_0 = \frac{\omega\omega_M}{\omega_0^2 - \omega^2} \tag{37}$$

$$\chi_1 = -\frac{2\omega^2\omega_0\omega_M}{\left(\omega_0^2 - \omega^2\right)^2} \tag{38}$$

$$\kappa_1 = -\frac{\omega\omega_M\left(\omega_0^2 + \omega^2\right)}{\left(\omega_0^2 - \omega^2\right)^2}. \tag{39}$$

These quantities are defined such that $\mu_{\text{eff}} = \mu_{\text{eff}}^{(0)} + (\beta V/\omega)\mu_{\text{eff}}^{(1)}$, $\chi' = \chi_0 + (\beta V/\omega)\chi_1$, and $\kappa' = \kappa_0 + (\beta V/\omega)\kappa_1$ to first order in $(\beta V/\omega)$. Note that the higher order terms in μ_{eff} are powers of V/u rather than V/c. Since $u \ll c$ for magnetostatic waves, it is possible that under some circumstances it is valid to retain the nonlinear terms in μ_{eff} while still neglecting nonlinear terms in V/c. In most cases of practical interest, however, the difference between (31) and (33) is negligible. Since the explicit elimination of higher powers of V/u results in more complicated expressions, higher order terms will be dropped only when doing so simplifies the result.

When $\alpha = 0$, (31) and (33) give the dispersion relations for uniform plane waves in an infinite moving ferrite. For both α and V equal to 0, the ordinary and extraordinary wave dispersion relations for an infinite stationary ferrite are recovered [10]. The ordinary wave is found to be a TEM wave with the magnetic field parallel to the z axis.

Because the medium is magnetically saturated along this direction, there is no small signal susceptibility and hence no magnetic interaction with the medium. In contrast, the extraordinary wave is a TE mode with components of the magnetic field both along x and y. Since the medium does exhibit small signal susceptibilities along these directions, the extraordinary wave is characterized by a strong magnetic interaction with the ferrite.

The general expression for nonuniform plane waves (eq. (31)) will be needed in subsection D to obtain the surface modes of a thin ferrite film.

C. Field Solutions for Extraordinary Nonuniform Plane Waves

To find the electric field components for extraordinary nonuniform plane waves, we must examine (26) where the wave matrix is obtained by substituting the appropriate form of the dispersion relation (31) into (28). From this it is found that the only nonvanishing component of the electric field intensity is E_z. Assuming $\boldsymbol{E}$ is known, the magnetic field intensity can be calculated from

$$\boldsymbol{H}=\frac{1}{\omega}\bar{\mu}^{-1}\cdot\left[\boldsymbol{k}\times\bar{\boldsymbol{I}}-\omega\bar{\xi}\right]\cdot\boldsymbol{E}. \tag{40}$$

This result is obtained by combining Maxwell's curl $\boldsymbol{E}$ equation with the constitutive law (15). Substituting $\boldsymbol{E}=\hat{z}E_z$ into (40) shows that H_x and H_y are the nonvanishing components of the magnetic field. Because there is only one electric field component but two magnetic field components, we choose to normalize to E_z. Thus if the electric field is specified by

$$E_{zs}=C_s e^{s\alpha x+i\beta y} \tag{41}$$

the magnetic field is found to be

$$H_{xs}=\frac{1}{\omega\mu_0}Q_sC_se^{s\alpha x+i\beta y} \tag{42}$$

$$H_{ys}=\frac{-i}{\omega\mu_0}R_sC_se^{s\alpha x+i\beta y} \tag{43}$$

where

$$Q_s=\frac{(1+\chi')[c^2\beta-\omega V[1-\epsilon_r(1+\chi')]]-\kappa'[s\alpha c^2+\omega V\epsilon_r\kappa']}{c^2[(1+\chi')^2-(\kappa')^2]} \tag{44}$$

and

$$R_s=\frac{\kappa'[c^2\beta-\omega V[1-\epsilon_r(1+\chi')]]-(1+\chi')[s\alpha c^2+\omega V\epsilon_r\kappa']}{c^2[(1+\chi')^2-(\kappa')^2]}. \tag{45}$$

The above forms emphasize the symmetry between R_s and Q_s. An algebraically simplified form for R_s is given below in (62).

D. Surface Modes of a Thin Ferrite Film

We now turn our attention to the layered structure shown in Fig. 2. The layers on either side of the ferrite are assumed to be nonmagnetic dielectrics. Following the usual approach for boundary value problems, we will write down general expressions for the guided mode fields in each region and then match boundary conditions.

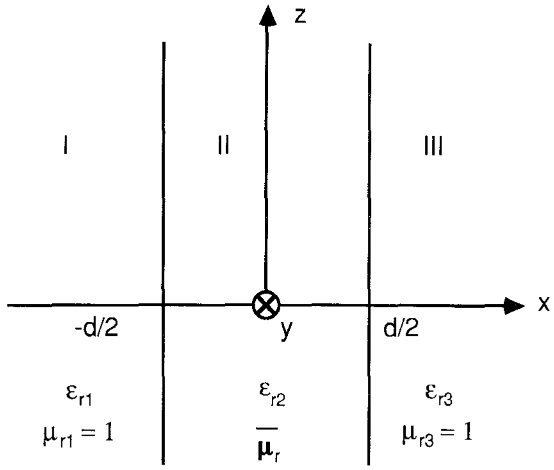

Fig. 2. Ferrite thin film geometry.

The fields in the dielectric region I can be obtained from (41)–(45) by setting $\chi'=\kappa'=0$ and choosing $s=+1$ since decaying solutions are needed as $x\to-\infty$. Thus, for region I:

$$H_{x1}=\frac{1}{\omega\mu_0}\left[\beta-\frac{\omega V}{c^2}(1-\epsilon_{r1})\right]C_1e^{\alpha_1x+i\beta y} \tag{46}$$

$$H_{y1}=\frac{i\alpha_1}{\omega\mu_0}C_1e^{\alpha_1x+i\beta y} \tag{47}$$

$$E_{z1}=C_1e^{\alpha_1x+i\beta y}. \tag{48}$$

In region II, the general solution is composed of both growing and decaying terms:

$$H_{x2}=\frac{1}{\omega\mu_0}\left[Q_+C_+e^{\alpha_2x}+Q_-C_-e^{-\alpha_2x}\right]e^{i\beta y} \tag{49}$$

$$H_{y2}=-\frac{i}{\omega\mu_0}\left[R_+C_+e^{\alpha_2x}+R_-C_-e^{-\alpha_2x}\right]e^{i\beta y} \tag{50}$$

$$E_{z2}=\left[C_+e^{\alpha_2x}+C_-e^{-\alpha_2x}\right]e^{i\beta y} \tag{51}$$

where R_s and Q_s are defined as in (44) and (45) with $\alpha=\alpha_2$ and $\epsilon_r=\epsilon_{r2}$, and the C's are constants to be determined.

Finally, the fields in region III are obtained by setting $\chi'=\kappa'=0$ and $s=-1$:

$$H_{x3}=\frac{1}{\omega\mu_0}\left[\beta-\frac{\omega V}{c^2}(1-\epsilon_{r3})\right]C_3e^{-\alpha_3x+i\beta y} \tag{52}$$

$$H_{y3}=-\frac{i\alpha_3}{\omega\mu_0}C_3e^{-\alpha_3x+i\beta y} \tag{53}$$

$$E_{z3}=C_3e^{-\alpha_3x+i\beta y}. \tag{54}$$

Since the motion is parallel to the boundaries, the boundary conditions are the same as for stationary media [11]. Thus we require tangential $\boldsymbol{E}$ and $\boldsymbol{H}$ to be continuous at $x = \pm d/2$. Applying the boundary condition on the electric field gives

$$C_1 e^{-\alpha_1 d/2} = C_+ e^{-\alpha_2 d/2} + C_- e^{\alpha_2 d/2} \quad (55)$$

$$C_3 e^{-\alpha_3 d/2} = C_+ e^{\alpha_2 d/2} + C_- e^{-\alpha_2 d/2}. \quad (56)$$

Similarly, requiring H_y to be continuous yields

$$-\alpha_1 C_1 e^{-\alpha_1 d/2} = R_+ C_+ e^{-\alpha_2 d/2} + R_- C_- e^{\alpha_2 d/2} \quad (57)$$

$$\alpha_3 C_3 e^{-\alpha_3 d/2} = R_+ C_+ e^{\alpha_2 d/2} + R_- C_- e^{-\alpha_2 d/2}. \quad (58)$$

Eliminating C_1 and C_3 from (55)–(58) gives a homogeneous linear system in the unknown amplitudes C_+ and C_-. The nontrivial solution is obtained by setting the determinant of the coefficient matrix to zero. The result is

$$e^{2\alpha_2 d} = \frac{(\alpha_1 + R_+)(\alpha_3 - R_-)}{(\alpha_1 + R_-)(\alpha_3 - R_+)}. \quad (59)$$

This is the dispersion relation in the laboratory frame for electromagnetic surface waves propagating in a moving thin film. For convenience, the parameter definitions are repeated below.

$$\alpha_i^2 = \beta^2 - \frac{\omega^2 \epsilon_{ri} \mu_{\text{eff},i}}{c^2} - \frac{2\omega\beta V}{c^2}[1 - \epsilon_{ri}\mu_{\text{eff},i}] \quad (60)$$

$$\mu_{\text{eff},i} = \mu_{\text{eff},3} = 1 \qquad \mu_{\text{eff},2} = \frac{(1+\chi')^2 - (\kappa')^2}{1+\chi'} \quad (61)$$

$$R_s = \frac{c^2[\kappa'\beta - s\alpha_2(1+\chi')] - \omega\kappa' V}{c^2\left[(1+\chi')^2 - (\kappa')^2\right]}. \quad (62)$$

When the medium velocity V is set to zero and media 1 and 3 are taken to be air, (59) reduces to the dispersion relation for electromagnetic surface waves in a ferrite layer obtained by Gerson and Nadan [12]. Electromagnetic waves guided by ferrite layers have also been discussed for stationary geometries by Karsono and Tilley [13], Marchand and Caillé [14], and Caillé and Thibaudeau [15]. These authors refer to the modes by the descriptive term *magnetic polaritons*.

Let us define $\Delta\beta$ as the change in wavenumber caused by the motion of the medium as observed in the laboratory frame; i.e., $\Delta\beta = \beta(\omega, V) - \beta(\omega, 0)$. This quantity is plotted in Fig. 3 for a medium velocity of $V = 10$ m/s and a typical yttrium iron garnet thin film (see the figure caption for the film parameters). For large values of β, the shift is much larger than would be expected on the basis of a simple Galilean velocity addition estimate. As will be emphasized in Section IV, this anomalously large shift is the result of the highly dispersive nature of the ferrite medium.

IV. Magnetostatic Limit

The magnetostatic limit is obtained by taking the limit $c \to \infty$ in (59)–(62). This gives

$$\alpha_i^2 = \beta^2 \quad (63)$$

$$R_s = \frac{\beta[\kappa' - s(1+\chi')]}{(1+\chi')^2 - (\kappa')^2}. \quad (64)$$

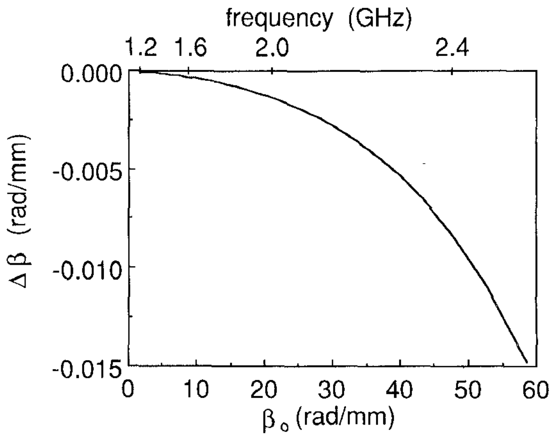

Fig. 3 Shift in wavenumber as observed in the laboratory frame caused by a film velocity of 10 m/s. Other parameters for the calculation are $H_0 = 6.37$ kA/m (80 Oe), $M_s = 140$ kA/m (1760 G), $\gamma_g/2\pi = 28$ GHz/T, $d = 15$ μ, $\epsilon_{r1} = 1$, $\epsilon_{r2} = 17$, and $\epsilon_{r3} = 12$.

Substituting these equations and the definitions of κ' and χ' from (21) and (22) and simplifying leads to the result

$$(\omega')^2 = \left[\omega_0 + \frac{\omega_M}{2}\right]^2 - \frac{\omega_M^2}{4} e^{-2\beta d}. \quad (65)$$

This is just the Damon and Eshbach dispersion relation [16] for magnetostatic surface waves but with the Doppler-shifted frequency $\omega' = \omega - \beta V$. This is reasonable since the magnetoelectric tensors (19) and (20) vanish in the limit $c \to \infty$, leaving the Doppler shift (25) as the only first-order effect of the motion. Writing $\beta = \beta_0 + \Delta\beta$ and explicitly keeping only first-order terms leads to the expression

$$\Delta\beta = -\frac{4\omega\beta_0 V e^{2\beta_0 d}}{\omega_M^2 d} \quad (66)$$

where β_0 satisfies (65) with $V = 0$. Equation (66) can be written more compactly in terms of the wave group velocity for a stationary medium. Setting $\omega' = \omega$ in (65) and differentiating with respect to β_0 gives

$$u_{g0}(\omega) = \frac{\omega_M^2 d}{4\omega} e^{-2\beta_0 d}. \quad (67)$$

Substituting this into (66) gives

$$\Delta\beta = -\beta_0 \frac{V}{u_{g0}}. \quad (68)$$

Thus the fractional change in the wavenumber is equal in magnitude to the ratio of the medium velocity to the wave group velocity in a stationary medium. Although derived specifically for the case of magnetostatic surface waves, it is shown in the Appendix that (68) is valid for all magnetostatic modes. The anomalously large values of $\Delta\beta$ observed in the previous section are explained by the fact that the wave group velocity goes asymptotically to 0 for large β_0. Equation (68) is indistinguishable from the full

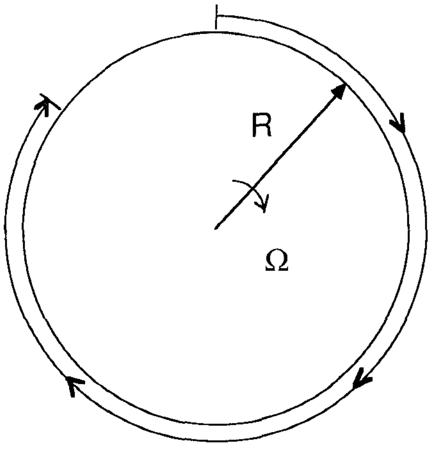

Fig 4. Geometry for analyzing the relative rotation rate sensor.

electromagnetic solution from Section III on the scale of Fig. 3.

It is also useful to obtain the magnetostatic limit of α_{FF} from (13). Taking the limit $c \to \infty$ yields

$$\alpha_{FF} = \frac{u_0(\omega)}{u_{g0}(\omega)}. \tag{69}$$

Although both the phase and group velocities of the magnetostatic waves vanish in the limit of arbitrarily large β_0, the group velocity vanishes more quickly. As a result, α_{FF} can be much larger than unity owing to the highly dispersive nature of the ferrite film. It is straightforward to show that (69) is equivalent to (68).

V. Relative Rotation Rate Sensing

As proposed by Newburgh *et al.* [1], magnetostatic surface waves guided around a circular path could be used to measure the relative rotation rate between the medium guiding the wave and an observer. The operation of such a sensor can be understood with the aid of Fig. 4. It is assumed that the wave is confined to a narrow region near the perimeter of the disk with radius R and propagates only in the clockwise (CW) direction (such nonreciprocal behavior is characteristic of magnetostatic surface waves [16]). If the radius R is large compared with the width of the confinement region, the results of the previous section can be used to estimate the change in phase caused by the rotation of the disk.[1] If the disk rotates CW with angular velocity Ω, the change in phase will be

$$\Delta\phi = \eta 2\pi R\,\Delta\beta = -\frac{2\eta A\beta_0}{u_{g0}}\Omega \tag{70}$$

where η is the path length expressed as a fraction of the circumference and A is the area enclosed by the path. Using typical values of $\beta_0 = 100$ cm^{-1}, $A = 3$ cm^2, $u_{g0} = 2.57\times10^7$ cm/s, $\Omega = 100\ \pi$ rad/s (3000 rpm), and $\eta = 1$ gives a phase shift of 0.42°. For $\beta_0 = 400$ cm^{-1}, $u_{g0} = 7.41\times10^6$ cm/s, $\Delta\phi$ becomes 5.83°.

The original argument in favor of magnetostatic and acoustic rotation rate sensors was greater sensitivity than that obtained with optical sensors based on the Sagnac effect [1]. According to this argument, the enhanced sensitivity results from the comparatively small wavelengths and phase velocities of these waves. However, as also pointed out by Newburgh *et al.* [1], the optical sensors are capable of sensing *absolute* rotation rates whereas the increased sensitivity for the alternative devices was predicted only for *relative* rotation rate sensing. Hence magnetostatic wave relative rotation rate sensors should not be compared with optical Sagnac devices, but with existing rotary tachometers. Also, the results of this section show that the magnitude of the phase shift is determined not by the wavelength and phase velocity alone, but by the resulting Doppler frequency shift combined with the dispersive nature of the ferrite. The usefulness of magnetostatic waves for *absolute* rotation rate sensing is discussed in the next section.

Briefly, there are three types of rotary tachometers in common use: the dc tachometer, ac tachometer, and digital pulse tachometer. In contrast with these devices, which produce output signal levels measured in volts, the proposed magnetostatic wave device would require the detection of small velocity dependent effects. Thus when compared with existing tachometers, enhanced sensitivity no longer seems a valid argument in favor of the magnetostatic wave device. However, both dc and ac tachometers suffer from the presence of rotation subharmonic ripple caused by the discrete number of armature windings. The addition of time constants to filter out these ripples can significantly affect the stability of high-performance systems. As an example, consider a dc tachometer with M commutations per revolution. The minimum time necessary to determine the average dc output is one period of the superimposed ripple. Thus the minimum response time (i.e., the lower limit on the time required to obtain the desired measurement) is estimated by

$$\tau_1 = \frac{1}{MS} \tag{71}$$

where S is the rotation rate in rev/s.

Digital tachometers produce a fixed number of output pulses per revolution. These pulses can be produced mechanically, magnetically, or optically. The rotation rate can be determined by counting the number of pulses in a given interval or by timing the duration of a single pulse.[2] Thus the minimum response time is estimated by

$$\tau_2 = \frac{1}{NS} \tag{72}$$

where N is the number of pulses per revolution.

[1] We also neglect intrinsic changes in the medium caused by the rotation, such as the Barnett effect [17].

[2] In practice, the duration of several pulses would have to be averaged to obtain a satisfactory result. The duration of a single pulse can be considered to be the minimum response time of an ideal device.

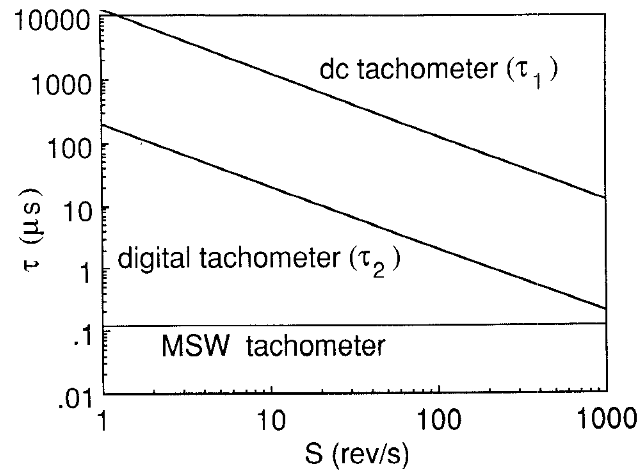

Fig. 5. Comparison of tachometer minimum response times. τ_1 and τ_2 were calculated from (71) and (72) respectively, with $M = 80$ and $N = 5000$. The magnetostatic wave response is based on propagation around a disk with $A = 3$ cm^2, $u_{g0} = 2.57 \times 10^7$ cm/s, and $\eta = 1$.

In contrast to the above devices, the minimum response time of the magnetostatic wave device does not vary with rotation rate but is determined by the time required for energy to propagate thorugh the device (i.e., the group delay). For the parameters in the previous numerical estimates, the response time would be about 400 ns or less.

Estimated minimum response times for conventional analog (dc) and digital tachometers as well as for the proposed magnetostatic wave device are compared in Fig. 5. (In practice, all the devices may require additional averaging to obtain the desired measurement accuracy.) The magnetostatic wave device has the potential for being significantly faster for rotation rates less than about 1000 rev/s (60000 rpm) for the particular parameters used. Such a device could contribute significantly to the stability of high-performance systems where high speed control is paramount.[3] Orientation-dependent effects from such things as mechanical wobble and magnetocrystalline anisotropy could, however, prevent this advantage from being realized [18], [19]. If the propagation path does not involve the entire circumference, then orientation-dependent effects will appear as different portions of the film are rotated into the propagation path. The effects can be due to changes in crystal orientation or to inhomogeneities in the film. It may be possible to minimize these effects by allowing the waves to propagate completely around the disk ($\eta = 1$). The phase shift should then not depend on orientation since no new portions of the film are brought into the path as the film is rotated.

VI. Absolute Rotation Rate Sensing

To measure absolute rotation rates, an interferometer is needed in which electromagnetic waves propagate in opposite directions around a ring. Owing to the Sagnac effect [20], there will be a phase difference between the two waves proportional to the rotation rate of the interferometer. Just as in the case of the Fresnel–Fizeau coefficient, the Sagnac effect can be described in terms of phase and group velocities rather than indices of refraction.

[3]Other factors such as the speed of the controller also affect system stability. The new device would therefore be most important for systems where the tachometers are the limiting factors.

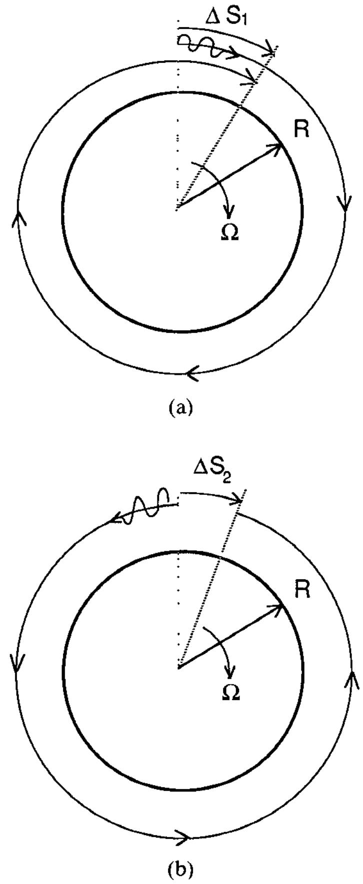

Fig. 6. Geometry for analyzing absolute rotation rate sensor. (a) Path of CW wave. (b) Path of CCW wave.

Referring to Fig. 6, consider a disk of radius R rotating with angular rate Ω containing waves propagating in both the clockwise (CW) and counterclockwise (CCW) directions. As before, we assume the energy is confined to a narrow region near the radius of the disk. In the laboratory (nonrotating) frame, the wavenumber of the CW wave is $\beta_0 - |\Delta\beta|$ and the wavenumber of the CCW wave is $\beta_0 + |\Delta\beta|$, where β_0 is the wavenumber in a stationary medium. After traveling around the ring and returning to the point at which they started, the phase shifts of the CW and CCW waves are

$$\phi_{CW} = [\beta_0 - |\Delta\beta|][2\pi R + \Delta S_1] \tag{73}$$

$$\phi_{CCW} = [\beta_0 + |\Delta\beta|][2\pi R - \Delta S_2]. \tag{74}$$

The phase difference between the two paths to lowest order in V/c is

$$\Delta\phi = \beta_0[\Delta S_1 + \Delta S_2] - 4|\Delta\beta|\pi R. \tag{75}$$

Since the ratio of the distances traveled by two objects in a time t is equal to the ratio of their velocities, ΔS_1 and ΔS_2

are approximated to lowest order by

$$\Delta S_1 \approx \Delta S_2 \approx \frac{\Omega R}{u_{g0}} 2\pi R. \tag{76}$$

The definition of α_{FF} in (1) can be used to express $\Delta\beta$ in terms of the medium velocity. The result to lowest order is

$$|\Delta\beta| = \alpha_{FF}\beta_0 \Omega R / u_0. \tag{77}$$

Combining (77) and (76) with (75) gives

$$\Delta\phi = \frac{4\beta_0 \Omega A}{u_{g0}} \left[1 - \alpha_{FF} \frac{u_{g0}}{u_0}\right] \tag{78}$$

where A is the area enclosed by the guided wave path. Substituting (13) for the Fresnel–Fizeau coefficient α_{FF} and simplifying gives the Sagnac phase shift for an interferometer with a corotating dispersive medium (i.e., a dispersive medium rotating with the interferometer):

$$\Delta\phi = \frac{4\omega\Omega A}{c^2}. \tag{79}$$

This is easily recognized as the Sagnac phase shift in the absence of a corotating medium. The fact that a corotating dielectric medium has no effect on the Sagnac phase shift has been pointed out by several authors [20], [21]. However, there is disagreement in the literature about the effect of a corotating magnetic medium. Yildiz and Tang [22] considered the case of a corotating scalar magnetic medium and concluded that the free-space phase shift (identical with (79)) should be multipled by the relative permeability. In contrast, Anderson and Ryon [23] considered the same case and concluded that the presence of a corotating scalar magnetic medium would have no effect on the phase shift. The results presented here agree with Anderson and Ryon and, in addition, should be valid for an arbitrary isotropic or anisotropic magnetic medium provided only that the phase and group velocities are collinear. To the author's knowledge, the disagreement regarding the effect of a magnetic medium has not been experimentally resolved. This is because Sagnac interferometers are usually constructed for optical frequencies, where magnetic effects are negligible. A microwave measurement using magnetostatic waves should make it possible to distinguish between the competing theories, however, owing to a large effective permeability.[4]

Newburgh *et al.* [1], have argued that an interferometer using magnetostatic surface waves could not be used to detect absolute rotations because of the way in which the wave velocity depends on the velocity of the medium. Indeed, if the magnetostatic form for α_{FF} (eq. (69)) is substituted into (78) the Sagnac phase shift vanishes. Rigorously speaking, however, magnetostatic waves are extraordinary electromagnetic waves and do exhibit the Sagnac effect when the correct form for α_{FF} is used. An optical interferometer is still preferable from a practical standpoint, however, because the large difference between microwave and optical frequencies results in optical phase shifts that are larger by four to five orders of magnitude than a microwave interferometer of the same area.

[4]The possibility of using microwave measurements to resolve this issue has also been pointed out by Post [24].

VII. Summary and Conclusions

The problem of waves propagating in moving ferrites has been considered from several viewpoints. First, the Fresnel–Fizeau drag coefficient was obtained for the case of a general anisotropic dispersive medium provided only that the phase and group velocities are collinear. The result was expressed in terms of phase and group velocities rather than an index of refraction to emphasize that its validity is not limited to optical frequencies.

Next, a first-order field theory for electromagnetic waves in moving ferrites was presented. The dispersion relation for nonuniform plane waves in an infinite medium was first derived, then applied to the boundary value problem of guided waves in a moving ferrite film. The magnetostatic limit was then extracted and shown to be in excellent agreement with the full electromagnetic theory. The dominant effect of the motion is found to be the Doppler-shifted frequency observed in the moving frame. This gives rise to an anomalously large shift in wavenumber owing to the dispersive nature of the ferrite medium. Because of the large effect, it is suggested that a moving medium experiment using magnetostatic waves could be used to distinguish between various competing forms for the dispersion term in the Fresnel–Fizeau coefficient.

Finally, the results of the field theory are applied to relative and absolute rotation rate sensing. In both applications the waves are considered to be confined to a narrow region near the edge of a rotating ferrite thin film. The large Fresnel–Fizeau coefficient suggests that magnetostatic waves could be used to measure relative rotation rates. Since the phase shift would be established in the time required to propagate around the disk, the response time could be significantly shorter than conventional tachometers. Although it should be possible, in principle, to construct an absolute rotation rate sensor using magnetostatic waves (or, more precisely, magnetic polaritons), the magnitude of the Sagnac effect is the same for magnetostatic waves as for ordinary electromagnetic waves with the same frequency. Since the magnitude of the Sagnac phase shift is proportional to frequency, optical interferometers are still preferable because of the much higher frequency.

Appendix

In general, the dispersion relation for magnetostatic waves can be written in the form

$$F(\omega, \beta, V) = 0. \tag{A1}$$

The solution to this equation can be approximated by a Taylor series for small V:

$$\beta(\omega, V) = \beta(\omega, 0) + \left.\frac{\partial\beta}{\partial V}\right|_{V=0} V. \tag{A2}$$

Thus

$$\Delta\beta = \left.\frac{\partial\beta}{\partial V}\right|_{V=0} V. \tag{A3}$$

The quantity $\partial\beta/\partial V$ can be obtained from (A1) by implicit differentiation:

$$\frac{\partial\beta}{\partial V} = -\frac{\partial F/\partial V}{\partial F/\partial\beta}. \tag{A4}$$

To evaluate $\partial F/\partial V$, we now make the assumption that the dependence of F on ω and V is of the form $F(\omega, V) = F(\omega - \beta V)$. As discussed in Section IV, this is valid for all magnetostatic modes propagating parallel to the motion of the medium. Thus

$$\frac{\partial F}{\partial V} = \frac{\partial F}{\partial\omega'}\frac{\partial\omega'}{\partial V} = -\beta\frac{\partial F}{\partial\omega'} \tag{A5}$$

where $\omega' = \omega - \beta V$. Substituting this result into (A4) and simplifying gives

$$\frac{\partial\beta}{\partial V} = -\beta\frac{\partial\beta}{\partial\omega'} = -\beta\frac{\partial\beta}{\partial\omega}\frac{\partial\omega}{\partial\omega'} = -\frac{\beta}{u_g}. \tag{A6}$$

Substituting this result into (A3) yields (68), as desired.

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